

三角公式大全

两角和公式

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \quad S_{\pm} = S_C \pm C_S$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \quad C_{\pm} = C_C \mp S_S$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$\cot(\alpha \pm \beta) = \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \alpha}$$

倍角公式

$$\sin(2\alpha) = 2 \sin \alpha \cdot \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha$$

$$\tan(2\alpha) = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

3 倍角公式

$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$\sin 3\alpha = 4 \sin \alpha \sin(60^\circ - \alpha) \sin(60^\circ + \alpha)$$

$$\cos 3\alpha = 4 \cos \alpha \cos(60^\circ - \alpha) \cos(60^\circ + \alpha)$$

$$\tan 3\alpha = \tan \alpha \tan(60^\circ - \alpha) \tan(60^\circ + \alpha)$$

半角公式

$$\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

$$\cot\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 + \cos \alpha}{1 - \cos \alpha}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$$

和差化积公式：

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\tan \alpha + \tan \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}$$

积化和差公式:

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\sin \alpha \sin \beta = -\frac{1}{2} [\cos(\alpha + \beta) - \cos(\alpha - \beta)]$$

万能公式

$$\sin \alpha = \frac{2 \tan \left(\frac{\alpha}{2} \right)}{1 + \left(\tan \left(\frac{\alpha}{2} \right) \right)^2}$$

$$\cos \alpha = \frac{1 - \left(\tan \left(\frac{\alpha}{2} \right) \right)^2}{1 + \left(\tan \left(\frac{\alpha}{2} \right) \right)^2}$$

$$\tan \alpha = \frac{2 \tan \left(\frac{\alpha}{2} \right)}{1 - \left(\tan \left(\frac{\alpha}{2} \right) \right)^2}$$

辅助角公式

$$A \sin \alpha + B \cos \alpha = \sqrt{A^2 + B^2} \sin(\alpha + \varphi), \tan \varphi = \frac{B}{A}$$

其他公式

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$1 + \tan^2 \alpha = 1 / \cos^2 \alpha$$

$$1 \pm \sin(\alpha) = \left(\frac{\sin \alpha}{2} \pm \frac{\cos \alpha}{2} \right)^2$$